Systèmes optiques cohérents:
Apport du traitement numérique des signaux

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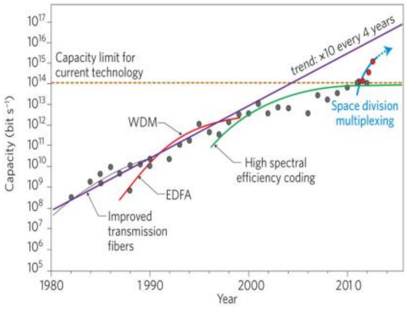
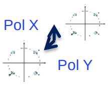
The Evolution of the Optical Fiber Capacity



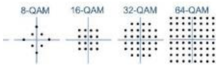
D.J. Richardson et al., "Space Division Multiplexing in optical fibers" Nature Photonics , 354-362, 2013.

- **1990s' EDFA → WDM technology**

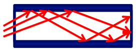
- **2000s' Coherent detection + Pol. Multiplexing**



- **2010s' High efficiency modulation formats**



- **Next: Space Division Multiplexing**
  - Multi-core fibers
  - Multimode fibers



Outline

- **Principles of optical coherent receiver**
- **Digital signal processing for coherent receivers**
  - Clock recovery
  - Equalization
    - *Deterministic equalization (chromatic dispersion, …)*
    - *Adaptative MIMO equalization (polarisation recovery, PMD, …)*
  - Laser phase noise mitigation
  - Frequency offset ($\Delta f_{S\text{-}Lo} \neq 0$)
- **Advanced DSP**
  - Non-unitary effects
  - Non-linear equalization

Optical Detection

Direct detection

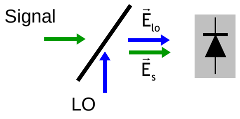
Photocurrent

$$I(t) \sim R\left|\vec{E}_s(t)\right|^2 \rightarrow I_{DD}(t) \sim RP_s(t)$$

Advantage : Low complexity implementation

Drawback : Limitation to OOK formats (NRZ or DPSK with MZI discriminator)

Coherent detection



Photocurrent

$$I(t) \sim R\left|\vec{E}_s + \vec{E}_{lo}\right|^2 \rightarrow I_{DC}(t) \sim R\vec{E}_s\vec{E}_{lo}$$

$$I_{DC}(t) \sim R\sqrt{P_s(t)}\underbrace{\sqrt{P_{lo}}}_{\text{sensivity}} \exp j\left(\underbrace{(\omega_s-\omega_{lo})t}_{\text{IF frequency}}+\varphi(t)\right)\underbrace{\vec{e}_s\vec{e}_{lo}}_{\text{Polarization alignment}}$$

Avantage : Access to complex envelope of optical signal $A_s(t) \sim \sqrt{P_s(t)} \exp j(\varphi(t))$

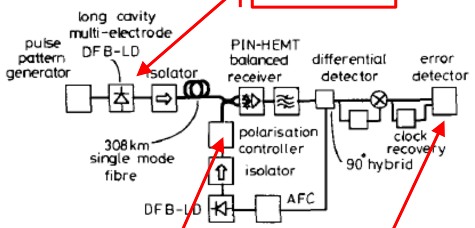
→ ***Application of I&Q formats such as xPSK, QAM, OFDM, ….***

Drawbacks : - Phase noise cancellation & Laser frequency stabilization
- Received signal is polarization dependant

Coherent Optical Systems in 90's

Tx: FSK (direct modulation → ~ some100MHz/mA)
Rx: Differential detection, No ADC, Threshold detection

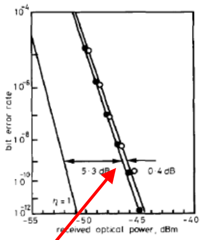
Experimentation 2.5 Gb/s (NTT)
Imai et al, Electronics Letters 1990



FSK modulation

Manual
Polarisation controler

OOK output



Sensitivity - 46.7dBm
(~67 photons/bit)

Sensitivity gain
~15 dB compared to DD

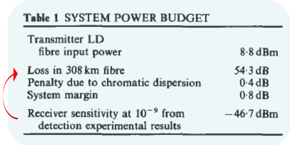


Table 1 SYSTEM POWER BUDGET

Transmitter LD fibre input power	8·8 dBm
Loss in 308 km fibre	54·3 dB
Penalty due to chromatic dispersion	0·4 dB
System margin	0·8 dB
Receiver sensitivity at 10^{-9} from detection experimental results	−46·7 dBm

Coherent Optical Systems in 90's

~~SNR sensivity~~

$$\cancel{SNR = \frac{2R^2 P_s P_{lo}}{\underbrace{2eRP_{lo}B_e}_{\text{Shot noise}} + \underbrace{4kTB_e / R_{ch}}_{\text{Thermal noise}}} \xrightarrow[P_{lo}\to\infty]{} \frac{\eta P_s}{h\nu B_e} = 2\eta\bar{N}}$$

$\cancel{\text{with } \bar{N} = \frac{P_s}{h\nu B_e}}$ *(photons number per symbol)*

Direct detection sensitivity

Without optical preamplification
$$SNR = \frac{(RP_s)^2}{4kTB_e / R_{ch}}$$

With optical preamplification
$$SNR = \frac{(RGP_s)^2}{4R^2 GP_s N_{ASE} B_e} \geq \eta\bar{N}/2$$

Optical preamplification allows to approach the quantum limit while maintaining the simplificy of direct detection (OOK format)

Increasing the Capacity

- **2000's: Deployment of 40Gb/s OOK systems**
  - High OSNR is required for distance > 1000km *(without Raman amplification !)*

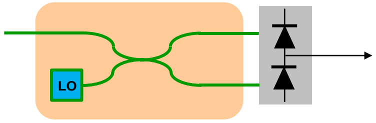
    SNR $= \frac{(RP_s)^2}{4R^2P_sN_{ASE}B_e} = \frac{P_s}{4N_{ASE}B_e}$ (OSNR $= \frac{P_s}{2N_{ASE}B_{0.1nm}}$)
  - PMD is a stochastic effect *(→ Optical PMD mitigation solution ?)*
- **Advantages of the coherent detection**
  - Higher SNR sensitivity **SNR** $= \frac{2R^2P_SP_{lo}}{4R^2P_{lo}N_{ASE}B_e} = \frac{2P_S}{4N_{ASE}B_e}$
  - SNR sensivity of the PSK is 3 dB higher than that in the ASK
  - Equalization in electrical domain (GVD & **PMD mitigation**)

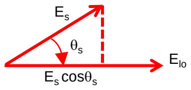
Fast ADC / DAC devices (ENOB 5/6bits, > 50Gsamples/s)
High speed DSP solution (FPGA/ASIC)

Coherent detection is now possible!

Phase-diversity Coherent Receiver

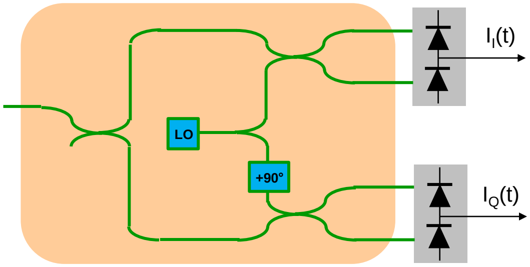
3 dB coupler (180° hybrid coupler

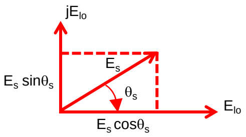




→ $E_s \cos\theta_s$
(only real part)

Phase-diversity coupler (90° hybrid coupler)

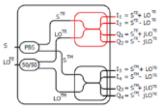




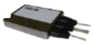
→ $E_s \exp j\theta_s$
(access to complex amplitude)

Phase-diversity Coherent Receiver

- **Advantages:**
  - The receiver is shot noise limited (balanced detection rejects DD components)
  - ASE noise is additive
  - Balanced detection allows linear detection
    - ⟶ *Standard DSP solutions, multi-level modulation formats*
  - Homodyne receiver is a baseband receiver
    - ⟶ *Ultra-selective channel filters with bandwidth ~ symbol rate*
- **Polarisation & phase-diversity receiver**



L = 70mm
l = 45mm
P = 10mm



Integrated coherent receiver

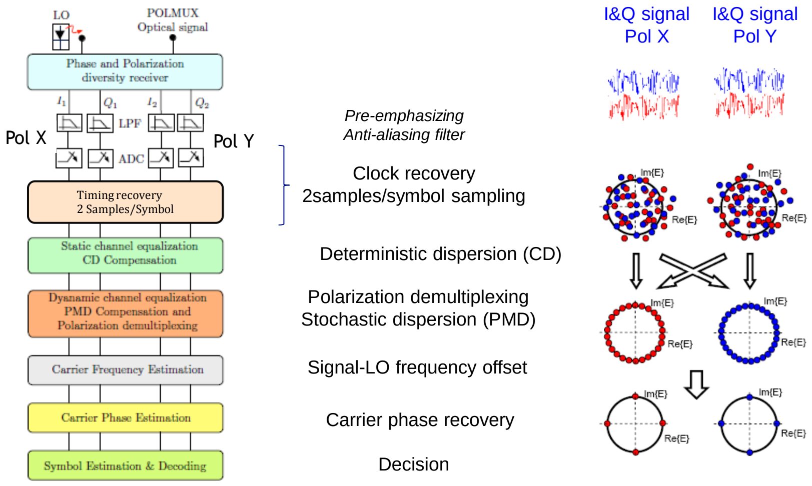
Outline

- Principle of optical coherent receiver
- **Digital signal processing in coherent receivers**
  - Clock recovery
  - Equalization
    - *Deterministic equalization (chromatic dispersion, …)*
    - *Adaptative MIMO equalization (polarisation recovery, PMD, …)*
  - Laser phase noise mitigation
  - Frequency offset ($\Delta f_{S\text{-}Lo} \neq 0$)
- Advanced DSP solutions
  - Non-unitary effects
  - Non-linear equalization

Physical Impairments

- **Fiber Propagation**
  - Chromatic dispersion (CD)
    - *quasi-deterministic effect*
    - ***TDE or FDE***
  - PMD
    - *Stochastic effect: PMD = <DGD>*
    - ***Adaptative 2x2 MIMO equalization***
  - Polarization-dependant loss (PDL)
    - *Current approach : OSNR margin*
    - *New approach : **space-time coding***
  - Nonlinear effects
    - *Intra-channel nonlinear equalizers (Digital backpropagation, Volterra series)*
    - ***Inter-band nonlinear interference cancelation***
- **Laser phase noise**
- **LO-signal frequency offset**

DSP Architecture



Pre-emphasizing
Anti-aliasing filter

Clock recovery
2samples/symbol sampling

Deterministic dispersion (CD)

Polarization demultiplexing
Stochastic dispersion (PMD)

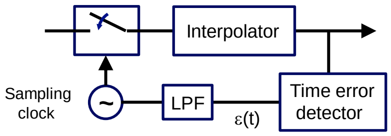
Signal-LO frequency offset

Carrier phase recovery

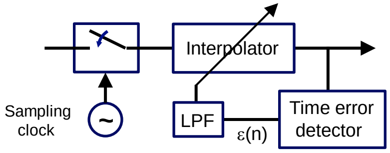
Decision

Clock Recovery

Analog-Digital timing recovery



Digital timing recovery



Gardner method (fully-digital)

- Used 2 samples per symbol
- Insentitive to carrier offset *(→ Timing recovery at front-end)*

$$\varepsilon(k) = Re\{x^*(kT_s - T_s/2 + \tilde{\tau}_{k-1})[x(kT_s + \tilde{\tau}_{k-1}) - x(kT_s - T_s + \tilde{\tau}_{k-1})]\}$$

- This timing error is passed througth a 2nd-order LPF such as

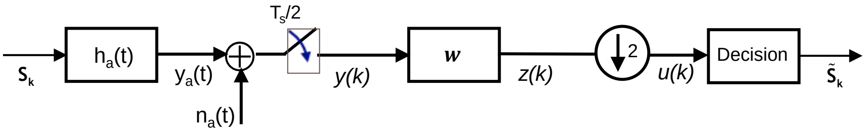
$\tau(k) = \lambda.\tau(k-1) + \ (1-\lambda).\varepsilon(k)$ *where λ is a forgotten factor*

The main purpose of Gardner algorithm is to place $x(kT_s - T_s/2)$ at the center of the consecutives symbols and to eliminate the error $\varepsilon(k)$

F. Gardner "A BPSK/QPSK timing-error detector for sampled receivers" IEEE Trans. Commun. 34, May 1986

Dispersion Effect

Channel model



{S$_k$} : transmitted symbols

h(t) : pulse shaping *(RRC filter)* + propagation effects *(GVD, PMD)*

Received signal $y_a(t) = h_a(t) \otimes S_k + n_a(t) \xrightarrow{sampling} y(k)$

Equalization $z(k) = \sum_j w(j)\, y(k-j) + n(k)$

Decision $\tilde{S}_k =$ decision [$u(k)$]

→ We find $w(t)$ that optimizes the performance

Dispersion Effect

Deterministic channel *(i.e. CD)*

→ Matched filter $w(t) = h(-t)^*$ → $W(\omega) = H(\omega)^*$

Unknown channel *(i.e PMD is a stochastic effect)*

We must find w(t) to minimize $E[|z(k) - S_k|^2]$

Constraint: $\{S_k\}$ is usually unknown

DD-LMS algorithm *(decision-directed)*

Phase 1 : Training mode (S_k known at the receiver)

→ w to minimize $E[|z(k) - S_k|^2]$

Phase 2 : Tracking mode, based on the most probable $\tilde{S}_k$ at Rx

→ w to minimize $E[|z(k) - \tilde{S}_k|^2]$

Constant Modulus Algorithm (CMA) -- *blind equalization*

For PSK, the modulus of $\{S_k\}$ is constant

→ w to minimize $E[(|z(k)|^2 - C)^2]$

D. Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication systems" Trans. Commun 28, Nov. 1980

Chromatic Dispersion

Dispersion effect

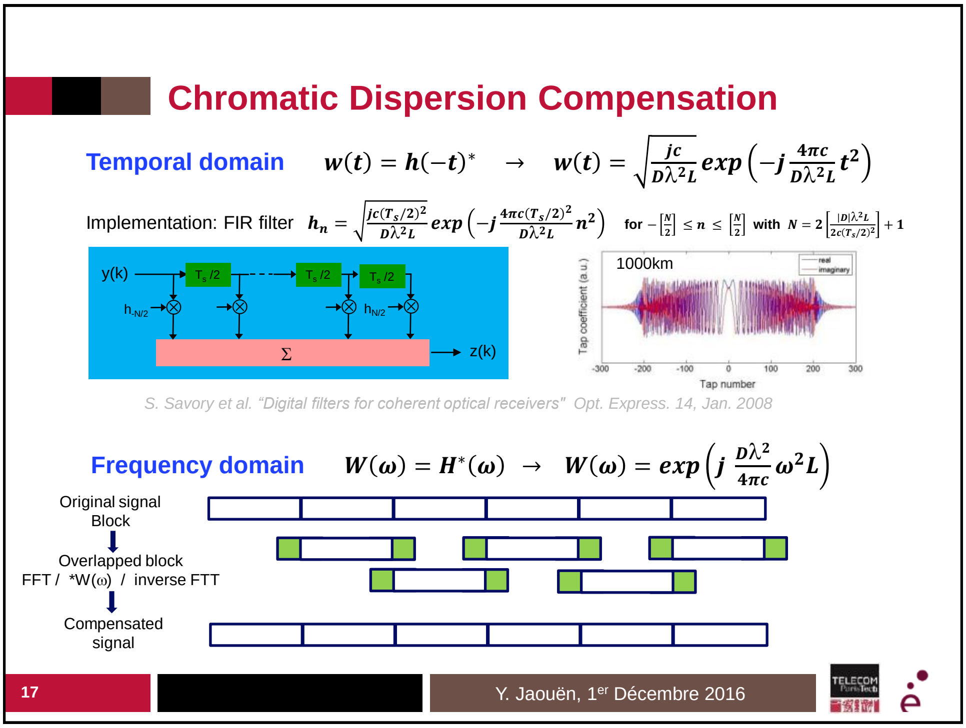
The refractive index is wavelength dependant → pulse broadening

$$\frac{\partial A(z,t)}{\partial z} = -j\frac{\beta_2}{2}\frac{\partial^2 A(z,t)}{\partial t^2} \xrightarrow{FT} \frac{\partial \tilde{A}(z,\omega)}{\partial z} = j\frac{\beta_2}{2}\omega^2\tilde{A}(z,\omega)$$

Channel model

Transfert function $H(\omega, L) = exp\left(-j\frac{D\lambda^2}{4\pi c}\omega^2 L\right)$

Impulse response $h(t, L) = \sqrt{\frac{c}{jD\lambda^2 L}}\, exp\left(j\frac{4\pi c}{D\lambda^2 L}t^2\right)$



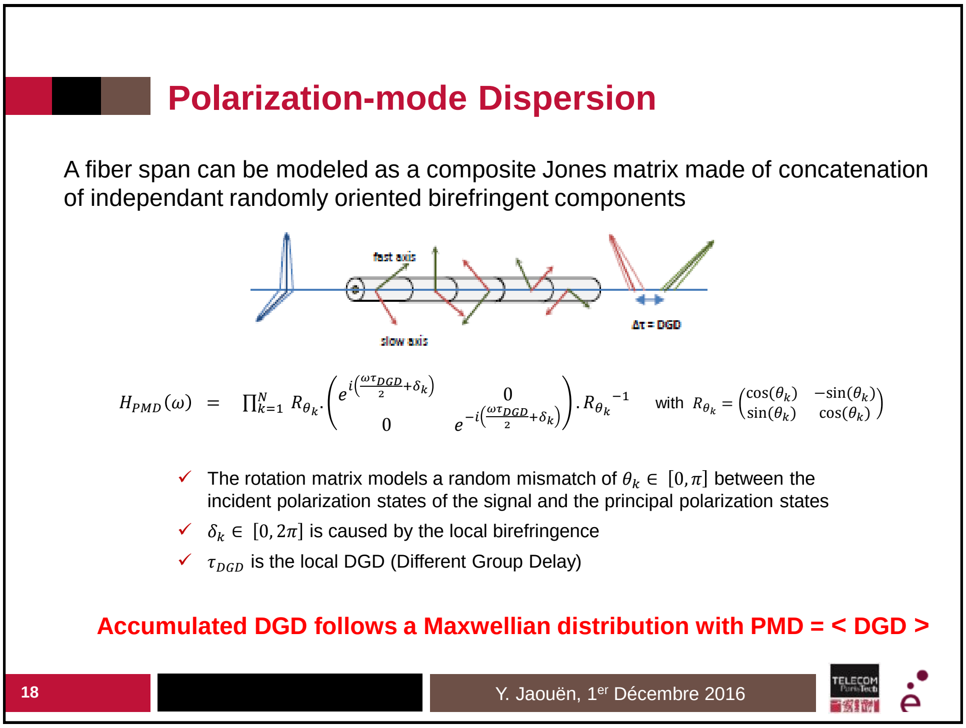
Chromatic Dispersion Compensation

Temporal domain $\boldsymbol{w(t) = h(-t)^*} \quad \rightarrow \quad \boldsymbol{w(t) = \sqrt{\frac{jc}{D\lambda^2 L}}\, exp\left(-j\,\frac{4\pi c}{D\lambda^2 L}\, t^2\right)}$

Implementation: FIR filter $\boldsymbol{h_n = \sqrt{\frac{jc(T_s/2)^2}{D\lambda^2 L}}\, exp\left(-j\,\frac{4\pi c(T_s/2)^2}{D\lambda^2 L}\, n^2\right)}$ **for** $-\left[\frac{N}{2}\right] \leq n \leq \left[\frac{N}{2}\right]$ **with** $N = 2\left[\frac{|D|\lambda^2 L}{2c(T_s/2)^2}\right] + 1$

S. Savory et al. *"Digital filters for coherent optical receivers" Opt. Express. 14, Jan. 2008*

Frequency domain $\boldsymbol{W(\omega) = H^*(\omega)} \quad \rightarrow \quad \boldsymbol{W(\omega) = exp\left(j\,\frac{D\lambda^2}{4\pi c}\,\omega^2 L\right)}$



Polarization-mode Dispersion

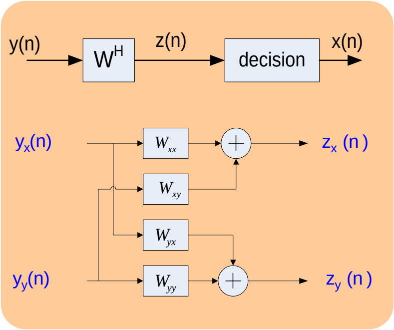
A fiber span can be modeled as a composite Jones matrix made of concatenation of independant randomly oriented birefringent components

$$H_{PMD}(\omega) \;=\; \prod_{k=1}^{N} R_{\theta_k}.\begin{pmatrix} e^{i\left(\frac{\omega\tau_{DGD}}{2}+\delta_k\right)} & 0 \\ 0 & e^{-i\left(\frac{\omega\tau_{DGD}}{2}+\delta_k\right)} \end{pmatrix}.{R_{\theta_k}}^{-1} \quad \text{with} \; R_{\theta_k} = \begin{pmatrix} \cos(\theta_k) & -\sin(\theta_k) \\ \sin(\theta_k) & \cos(\theta_k) \end{pmatrix}$$

- ✓ The rotation matrix models a random mismatch of $\theta_k \in [0, \pi]$ between the incident polarization states of the signal and the principal polarization states
- ✓ $\delta_k \in [0, 2\pi]$ is caused by the local birefringence
- ✓ τ_{DGD} is the local DGD (Different Group Delay)

Accumulated DGD follows a Maxwellian distribution with PMD = < DGD >

MIMO Equalization (1)



Equalizer W → 4 FIR filters

- 1 Tap : SOP recovery only
- 3 to ~15 Taps :
  SOP recovery + GVD & PMD compensation

Channel without PDL: W is **unitary matrice**

$$\begin{pmatrix} w_{xx} & w_{xy} \\ -w_{xy}^{*} & w_{xx}^{*} \end{pmatrix}$$

Two differents approaches:

- ✓ H is estimated using training symbols → ZF equalization *(W is pseudo-inverse of H)*
- ✓ Blind equalization → Gradient algorithm $W_{n+1} = W_n - \mu \nabla J_n|_{W_n}$
  where J is cost function

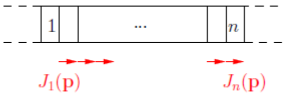
MIMO equalization (2)

$$W_{n+1} = W_n - \mu \nabla J_n\Big|_{W_n}$$

	Decision- Directed	CMA
Cost function	$J_{DD}(\mathbf{W})=\mathbf{E}\left(\left\|z(n)-\hat{x}(n)\right\|^2\right)$	$J_{CMA}(\mathbf{W})=\mathbf{E}\left(\left\|z(n)\right\|^2-R\right)^2$
$\nabla J_n(\mathbf{W})$	$(z(n)-\hat{x}(n))^{*}\mathbf{y}(n)$	$\left(\left\|z(n)\right\|^2-R\right)z(n)^{*}\mathbf{y}(n)$

How to find the minimum of $J(p) = \mathbf{E}[J_n(p)]$?

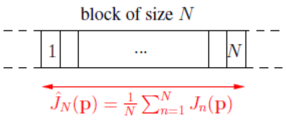
Adaptative processing



We replace $J(p)$ with J_n
at iteration n (stochastic gradient)

$$W_{n+1} = W_n - \mu \frac{\partial J_n}{\partial W}\Big|_{W_n}$$

Blockwise processing



We replace $J(p)$ with $\partial\hat{J}_N$

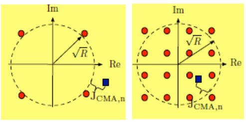
$$W_{n+1} = W_n - \mu \frac{\partial \hat{J}_N}{\partial W}\Big|_{W_n}$$

MIMO equalization (3)

- **Constant Modulus Algorithm (CMA)**
  - Error function

$$J_{CMA}(W_n) = (\,|z(n)|^2 - R)^2$$

  - CMA is not optimal for Multi Modulus formats

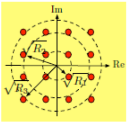


QPSK 16-QAM

- **Radius Directed Equalizer (RDE)**
  - Error function

$$J_{RDE}(W_n) = (\,|z(n)|^2 - R_i)^2, \quad R_i \in \{R_1, R_2, R_3\}$$

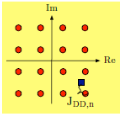
  - RDE requires pre-convergence using CMA



- **Decision-directed (DD)**
  - Error function

$$J_{DD}(W_n) = \left(\,|z(n)|^2 - \hat{S}(n)\right)^2$$

  - DD is carried out after CMA/RDE

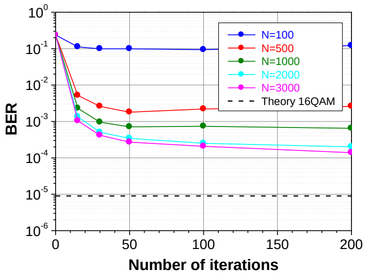


Numerical illustration

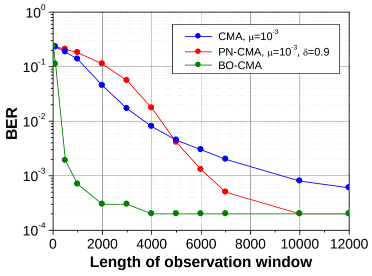
System Parameters

M. Selmi et al. "Block-wise digital signak processing for PolMux QAM/PSK optical coherent systems" J. Lightwave Technol., LT-29, Oct. 2011

- *PolMux 16-QAM, 112Gb/s*
- *OSNR = 20dB, CD = 1000ps/nm, DGD = 50ps, $\theta = \pi/4$)*

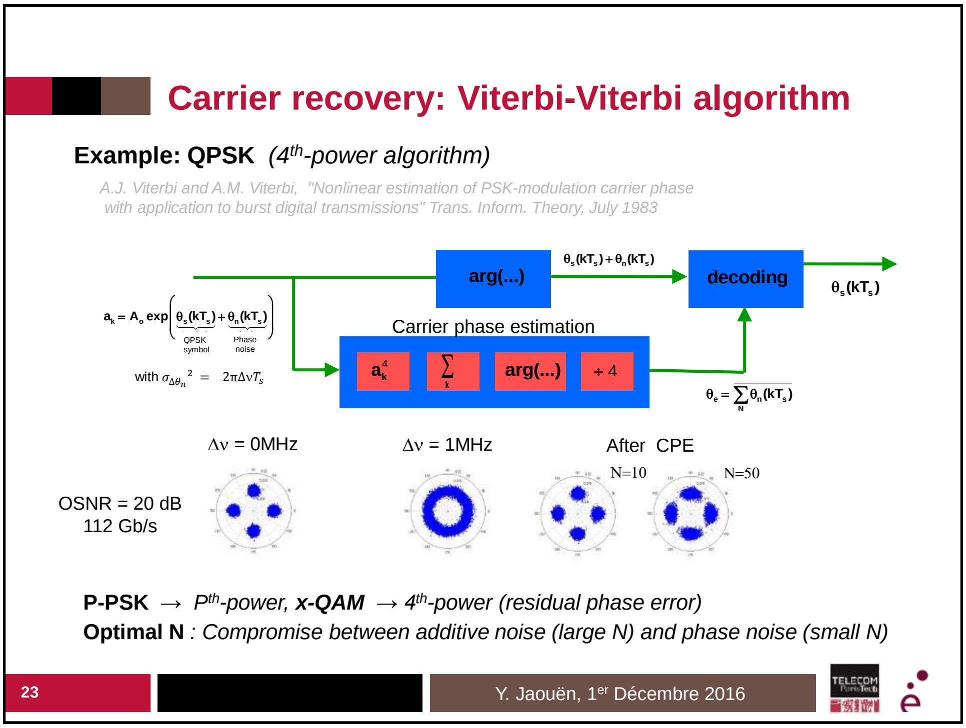


N = 1000 is a good compromise
Between convergence and block size



Block CMA has fastest convergence
compared to Adaptive CMA (~10000 samples)

(*) PN-CMA Pseudo-Newton $\mu \rightarrow \mu_n H_n^{-1}$



Carrier recovery: Viterbi-Viterbi algorithm

Example: QPSK *(4th-power algorithm)*

A.J. Viterbi and A.M. Viterbi, "Nonlinear estimation of PSK-modulation carrier phase with application to burst digital transmissions" Trans. Inform. Theory, July 1983

$$a_k = A_o \exp\left(\underbrace{\theta_s(kT_s)}_{\text{QPSK symbol}} + \underbrace{\theta_n(kT_s)}_{\text{Phase noise}}\right)$$

with $\sigma_{\Delta\theta_n}{}^2 = 2\pi\Delta\nu T_s$

OSNR = 20 dB
112 Gb/s

P-PSK → *Pth-power,* ***x-QAM*** → *4th-power (residual phase error)*

Optimal N *: Compromise between additive noise (large N) and phase noise (small N)*

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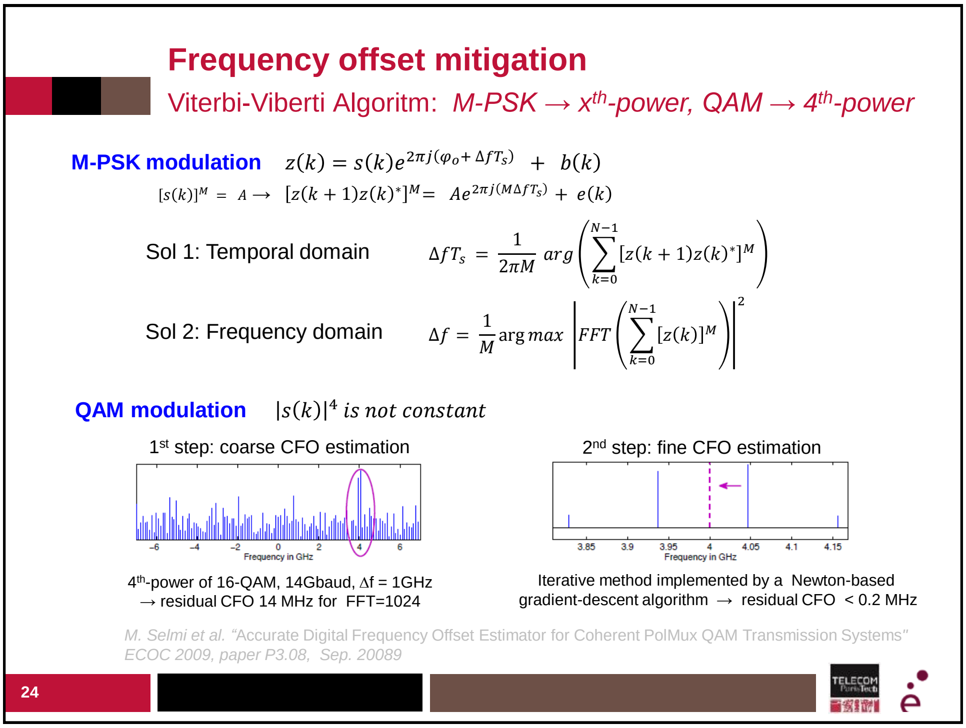
Y. Jaouën, 1er Décembre 2016



Frequency offset mitigation

Viterbi-Viberti Algoritm: *M-PSK → xth-power, QAM → 4th-power*

M-PSK modulation $z(k) = s(k)e^{2\pi j(\varphi_o + \Delta f T_s)} + b(k)$

$$[s(k)]^M = A \rightarrow [z(k+1)z(k)^*]^M = Ae^{2\pi j(M\Delta f T_s)} + e(k)$$

Sol 1: Temporal domain $$\Delta f T_s = \frac{1}{2\pi M} arg\left(\sum_{k=0}^{N-1}[z(k+1)z(k)^*]^M\right)$$

Sol 2: Frequency domain $$\Delta f = \frac{1}{M}\arg max\left|FFT\left(\sum_{k=0}^{N-1}[z(k)]^M\right)\right|^2$$

QAM modulation $|s(k)|^4$ *is not constant*

1st step: coarse CFO estimation

4th-power of 16-QAM, 14Gbaud, Δf = 1GHz
→ residual CFO 14 MHz for FFT=1024

2nd step: fine CFO estimation

Iterative method implemented by a Newton-based gradient-descent algorithm → residual CFO < 0.2 MHz

M. Selmi et al. "Accurate Digital Frequency Offset Estimator for Coherent PolMux QAM Transmission Systems" ECOC 2009, paper P3.08, Sep. 20089

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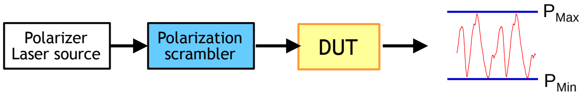
Outline

- Principle of optical coherent receiver
- Digital signal processing in coherent receivers
  - Clock recovery
  - Equalization
    - *Deterministic equalization (chromatic dispersion, …)*
    - *Adaptative MIMO equalization (polarisation recovery, PMD, …)*
  - Laser phase noise mitigation
  - Frequency offset ($\Delta f_{S\text{-}Lo} \neq 0$)
- **Advanced DSP solutions**
  - Non-unitary effects
  - Non-linear effects equalization

Polarization-dependent Loss (1)

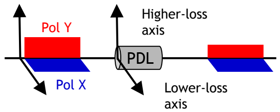
- PDL is introduced by optical elements along fiber length



$$PDL_{dB} = 10\log\left(\frac{P_{Max}}{P_{Min}}\right)$$

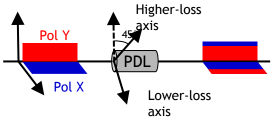
PDL corresponds to the peak-to-peak of the transmit signal

- Induced penalties
  - Case 1: PDL axes are aligned with polarization components of input signal



PDL causes extra-loss for one polarisation state resulting in OSNR difference
- *PDL cannot be compensated by DSP*
- *Capacity reduction*

  - Case 2: PDL axes are rotated by 45°



Same OSNR but loss of orthogonality
- *Crosstalk between the 2 polarization tributaries*
- *Possible mitigation by DSP*

For intermediate values, the polarization states will suffer a mix of these two impairments: OSNR inequalities and loss of orthogonality

Polarization-Dependent Loss (2)

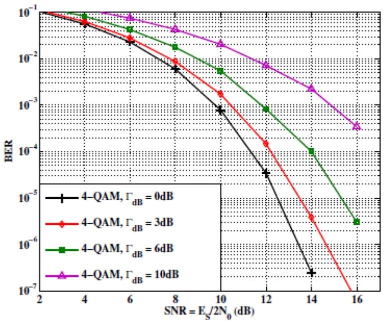
PDL depends on the SOP of the signal and thus is a stochastic process

The transfer matrix can be modeled by:

$$H_{\omega} = R_{\theta}\begin{pmatrix}\sqrt{1+\gamma} & 0 \\ 0 & \sqrt{1-\gamma}\end{pmatrix}R_{\theta}^{-1}$$

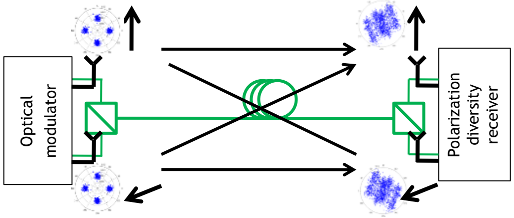
- The PDL is expressed as $\Gamma_{\mathrm{dB}} = 10\log\frac{1+\gamma}{1-\gamma}$
- R_{θ} and R_{θ}^{-1} are random rotation matrices with uniform distribution

4-QAM, Lumped PDL

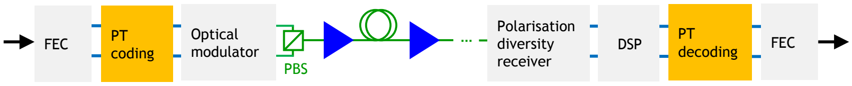


Polarization-Time Coding (1)

An optical PolMux system can be seen as a 2x2 MIMO system



Space-Time coding techniques can be applied on PolMux systems



Objective: take benefit of the polarization-multiplexed transmission

S. Mumtaz, G. Rekaya Ben Othman and Y. Jaouën, "PDL mitigation in PolMux OFDM systems using Golden and Silver Polarization-Time codes" OFC'10, paper JThA7, May 2010

Polarization-Time Coding (2)

Principle: send a linear combination of modulated symbols during many symbol durations on each polarization

Polarization-Time code codeword matrix

$$\mathbf{X} = \begin{array}{ccl} t_1 & t_2 & \\ f_1(S_1,S_2,S_3,S_4\ldots) & f_3(S_1,S_2,S_3,S_4\ldots) & \text{Polarization 1} \\ f_2(S_1,S_2,S_3,S_4\ldots) & f_4(S_1,S_2,S_3,S_4\ldots) & \text{Polarization 2} \end{array}$$

$$\mathbf{X} = \begin{bmatrix} X_{pol1,t_1} & X_{pol1,t_2} \\ X_{pol2,t_1} & X_{pol2,t_2} \end{bmatrix} =$$

$$\mathbf{X}_{\mathcal{G}} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\,(s_1+\theta s_2) & \alpha\,(s_3+\theta s_4) \\ i\bar{\alpha}\,(s_3+\bar{\theta} s_4) & \bar{\alpha}\,(s_1+\bar{\theta} s_2) \end{bmatrix} \quad \text{Golden code}$$

$$\theta = \frac{1+\sqrt{5}}{2}, \quad \bar{\theta} = \frac{1-\sqrt{5}}{2} \qquad \alpha = 1+i-i\theta, \quad \bar{\alpha} = 1+i-i\bar{\theta}$$

$$\mathbf{X}_{\mathcal{S}} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} z_1 & -z_2^* \\ z_2 & z_1^* \end{bmatrix} \quad \text{Silver code}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \frac{1}{\sqrt{7}} \begin{bmatrix} 1+i & -1+2i \\ 1+2i & 1-i \end{bmatrix} \begin{bmatrix} s_3 \\ s_4 \end{bmatrix}$$

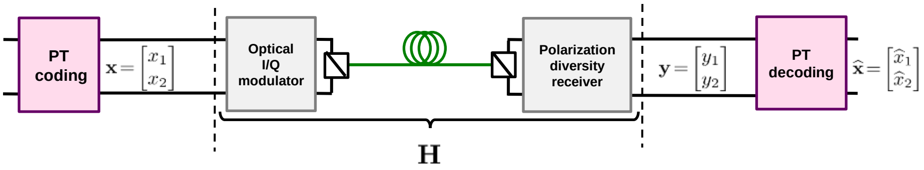
Full rate
→ 2 Symb/ ch use

$$\mathbf{X}_{\mathcal{A}} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \quad \text{Alamouti code}$$

No full rate
→ 1 Symb/ ch use

Polarization-time block code decoding

Single carrier and time-domain processing at reception



- The received symbols are: $\mathbf{Y} = \mathbf{HX} + \mathbf{n}_{\text{ASE}}$

2x2 H$_i$ represents the ith tap of impulse channel response

$$\begin{bmatrix} \mathbf{y}(k) \\ \mathbf{y}(k-1) \\ \mathbf{y}(k-2) \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \ldots & \mathbf{H}_L & 0 & 0 & \ldots & & \\ 0 & \mathbf{H}_1 & \mathbf{H}_2 & \ldots & \mathbf{H}_L & 0 & 0 & \ldots & \\ 0 & 0 & \mathbf{H}_1 & \mathbf{H}_2 & \ldots & \mathbf{H}_L & 0 & 0 & \ldots \\ & & & & \ddots & & & & \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k-1) \\ \mathbf{x}(k-2) \\ \vdots \end{bmatrix} + \mathbf{n}_{\text{ASE}}$$

- **Maximum Likelihood (ML) decoding**

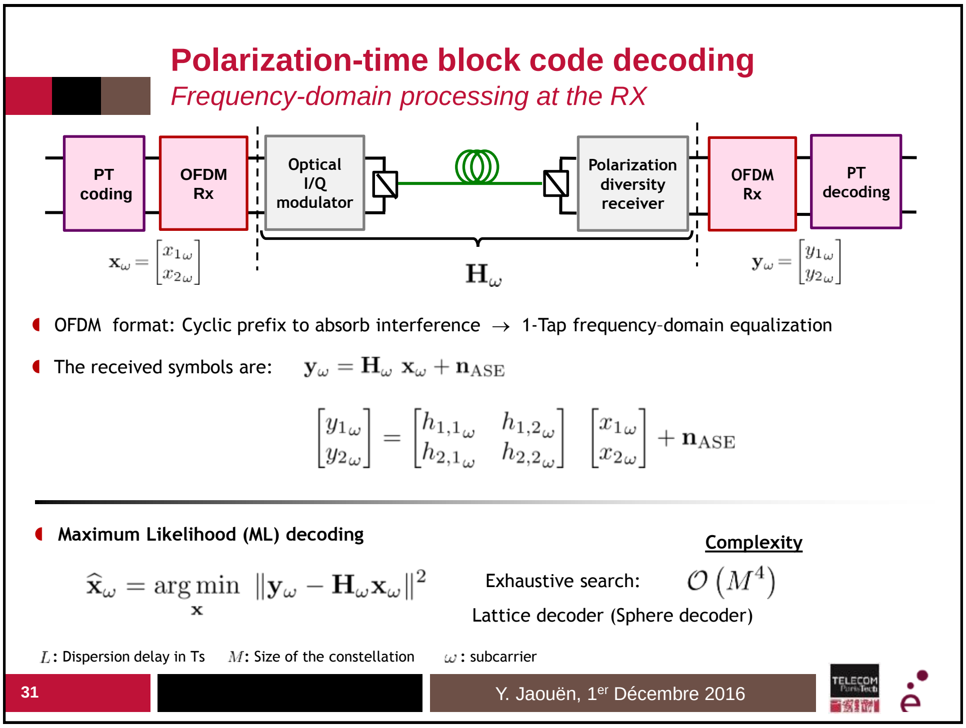
$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{Y} - \mathbf{HX}\|^2$$

Complexity

Exhaustive search: $\mathcal{O}\left(M^{2\times(\boxed{L}+1)}\right)$

Using Orthogonal frequency division multiplexing (OFDM) can remove dispersion effects and thus reduce the decoding complexity

L: Dispersion delay in Ts M: Size of the constellation ω: subcarrier



Polarization-time block code decoding

Frequency-domain processing at the RX

PT coding

OFDM Rx

Optical I/Q modulator

Polarization diversity receiver

OFDM Rx

PT decoding

$\mathbf{x}_\omega = \begin{bmatrix} x_{1\omega} \\ x_{2\omega} \end{bmatrix}$ $\quad \mathbf{H}_\omega \quad$ $\mathbf{y}_\omega = \begin{bmatrix} y_{1\omega} \\ y_{2\omega} \end{bmatrix}$

- OFDM format: Cyclic prefix to absorb interference → 1-Tap frequency-domain equalization
- The received symbols are: $\mathbf{y}_\omega = \mathbf{H}_\omega\ \mathbf{x}_\omega + \mathbf{n}_{\mathrm{ASE}}$

$$\begin{bmatrix} y_{1\omega} \\ y_{2\omega} \end{bmatrix} = \begin{bmatrix} h_{1,1\omega} & h_{1,2\omega} \\ h_{2,1\omega} & h_{2,2\omega} \end{bmatrix} \begin{bmatrix} x_{1\omega} \\ x_{2\omega} \end{bmatrix} + \mathbf{n}_{\mathrm{ASE}}$$

- **Maximum Likelihood (ML) decoding**

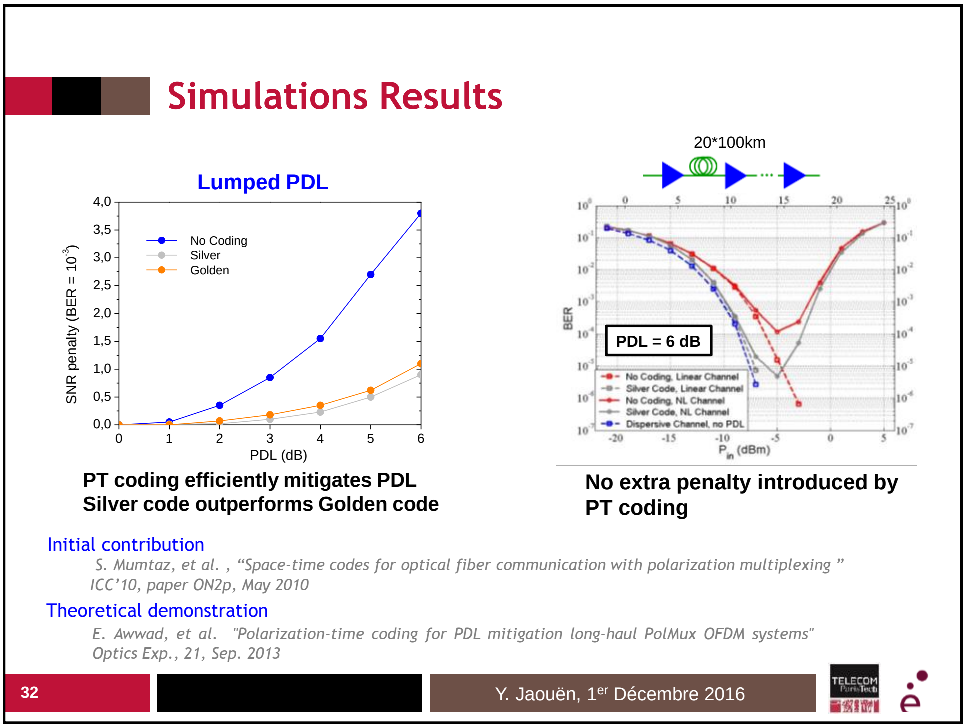
$$\widehat{\mathbf{x}}_\omega = \arg\min_{\mathbf{x}} \ \|\mathbf{y}_\omega - \mathbf{H}_\omega \mathbf{x}_\omega\|^2$$

Complexity

Exhaustive search: $\mathcal{O}\left(M^4\right)$

Lattice decoder (Sphere decoder)

L: Dispersion delay in Ts $\quad M$: Size of the constellation $\quad \omega$: subcarrier



Simulations Results

PT coding efficiently mitigates PDL
Silver code outperforms Golden code

No extra penalty introduced by PT coding

Initial contribution

S. Mumtaz, et al. , “Space-time codes for optical fiber communication with polarization multiplexing ” ICC’10, paper ON2p, May 2010

Theoretical demonstration

E. Awwad, et al. "Polarization-time coding for PDL mitigation long-haul PolMux OFDM systems" Optics Exp., 21, Sep. 2013

Nonlinearity Mitigation

- 400Gb/s transmission systems are based on modulation formats with high spectral efficiencies (16/64 QAM): higher OSNR is required
  - → More sensitive to nonlinear effects
- Multi-band approach to reach 400Gb/s - 1Tb/s
  - → Inter-band non-linear crosstalk
- Two main approaches have been considered for nonlinearity mitigation

1) Digital backpropagation (DBP)
   - Uses split-step Fourier Method (SSFM), many IFFTs/FFTs
   - High computational complexity
2) Inverse Volterra series transfer function (IVSTF)
   - Significantly lower computational complexity compared to multistep/span DBP
   - Lower computational time (parallel implementation)
   - Can be done in frequency domain avoiding aliasing phenomena

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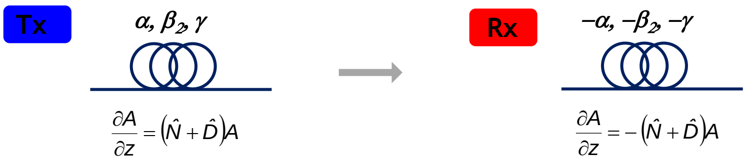
Nonlinearity Mitigation: *digital backpropagation*



$$\frac{\partial A}{\partial z} = (\hat{N} + \hat{D})A$$

$$\frac{\partial A}{\partial z} = -(\hat{N} + \hat{D})A$$

- **Numerical resolution based on split-step-Fourier method**
- **Deterministic nonlinear intra-channel interactions can be removed**
- **Achievable performances in WDM**

« Physical approach » (full spectrum bandwidth) → 1-2dB Q-factor improvement
  - → Very high complexity (*16/32 samples /symbol, Multi-step per span)*
  - → Used only as a reference for nonlinearity mitigation

« Electrical detected signal » (2 samples/symbol, 5-6 bits resolution, <10 steps/span)
  - → Single channel compensation (FWM/XPM ?)
  - → Typical Q-factor improvement limited to 0.5-1 dB

Nonlinearity Mitigation:
Inverse Volterra Series Transfer Function

- The NLS equation in time domain

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + j\gamma|A|^2 A$$

- The NLS equation in frequency domain

$$\frac{\partial \tilde{A}}{\partial z} = -\frac{\alpha}{2}\tilde{A} + j\frac{\beta_2}{2}\omega^2\tilde{A} + j\gamma\iint \tilde{A}(\omega_1,z)\tilde{A}^*(\omega_2,z)\tilde{A}(\omega-\omega_1+\omega_2,z)d\omega_1 d\omega_2$$

- 3th-order Volterra series expansion of fiber channel

$$\tilde{A}(\omega,z) = H_1(\omega,z)\tilde{A}(\omega,0) + \iint H_3(\omega,\omega_1,\omega_2,z)\tilde{A}(\omega_1,0)\tilde{A}^*(\omega_2,0)\tilde{A}(\omega-\omega_1+\omega_2,0)d\omega_1 d\omega_2$$

with

$$H_1(\omega,z) = \exp\left(-\frac{\alpha}{2}z\right)\exp\left(j\frac{\beta_2\omega^2 z}{2}\right)$$

$$H_3(\omega,z) = +j\gamma H_1(\omega,z)\frac{1-\exp(-\alpha z + j\beta_2(\omega_1-\omega)(\omega_1-\omega_2)z)}{\alpha - j\beta_2(\omega_1-\omega)(\omega_1-\omega_2)}$$

K. Peddanarappagari et al. "Volterra series transfer function of single-mode fibers" Lightwave Technology (1997)

Nonlinear equalizer can be designed based on VSTF

VSTF (α,β_2,γ) → IVSTF ($-\alpha,-\beta_2,-\gamma$)

Nonlinearity Mitigation:
Inverse Volterra Series Transfer Function

- IVSTF of an optically amplified multi-span fiber channel ***(without DCF)***

$$\tilde{A}(\omega,z) = H_1(\omega,z)A(\omega,0) + \iint H_3(\omega,\omega_1,\omega_2,z)\tilde{A}(\omega_1,0)\tilde{A}^*(\omega_2,0)\tilde{A}(\omega-\omega_1+\omega_2,0)d\omega_1 d\omega_2$$

with

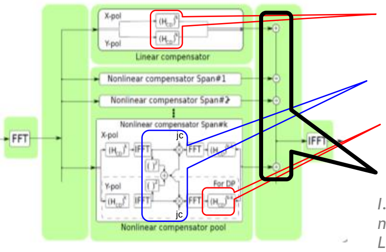
$$\begin{cases} K_1(\omega) = \exp\left(-j\frac{\beta_2\omega^2 NL}{2}\right) \\ K_3(\omega) = -j\gamma K_1(\omega)L_{eff}\sum_{k=0}^{N}\exp(-jk\beta_2\Delta\Omega L) \end{cases}$$

$$(\Delta\Omega=(\omega_1-\omega)(\omega_1-\omega_2))$$

$$L_{eff}=\frac{1-\exp(-\alpha L)}{\alpha}$$

L. *Liu et al.,"Intrachannel nonlinearity compensation by inverse Volterra series transfer function", J. Lightwave Technology, LT-30, 310-316 (2012)*

- Double Polarization IVSTF implementation



The linear compensation is simply a CD compensator

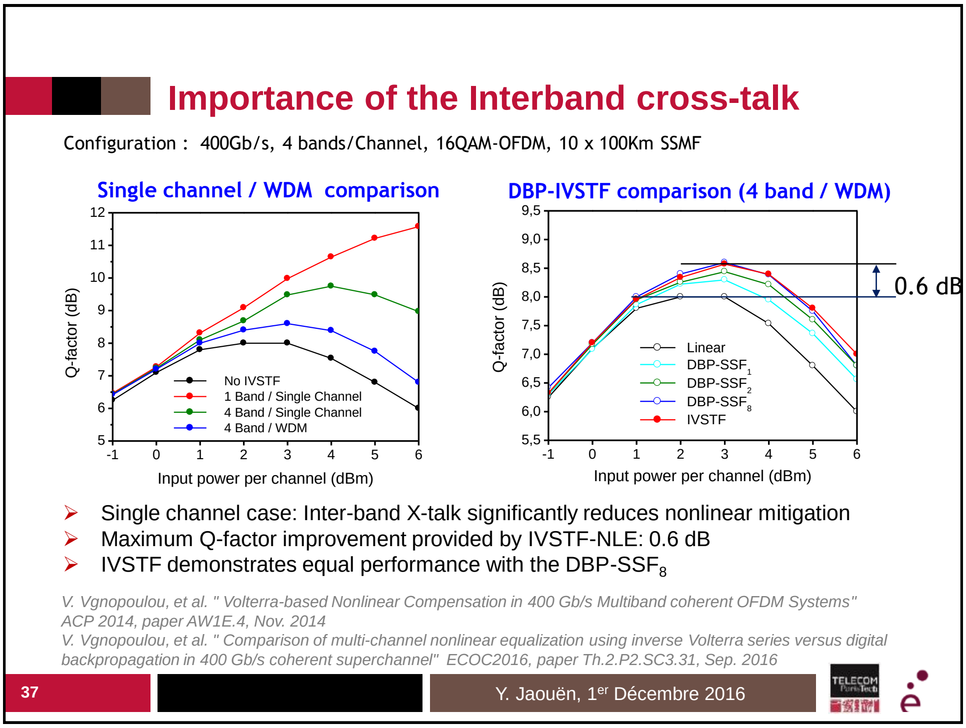
$$H_{CD} = \exp(-j\beta_2\omega L/2)$$

The nonlinear compensation is divided into N stages , where N is the span number $c \sim -\gamma L_{eff}$

Linear compensation of residual dispersion in frequency domain

Parallel implementation

I. Giacoumidis, et al. " Volterra-based reconfigurable nonlinear equalizer for coherent OFDM" Photon. Technol. Lett., PTL-26, pp. 1383-1886, July 2014.



Importance of the Interband cross-talk

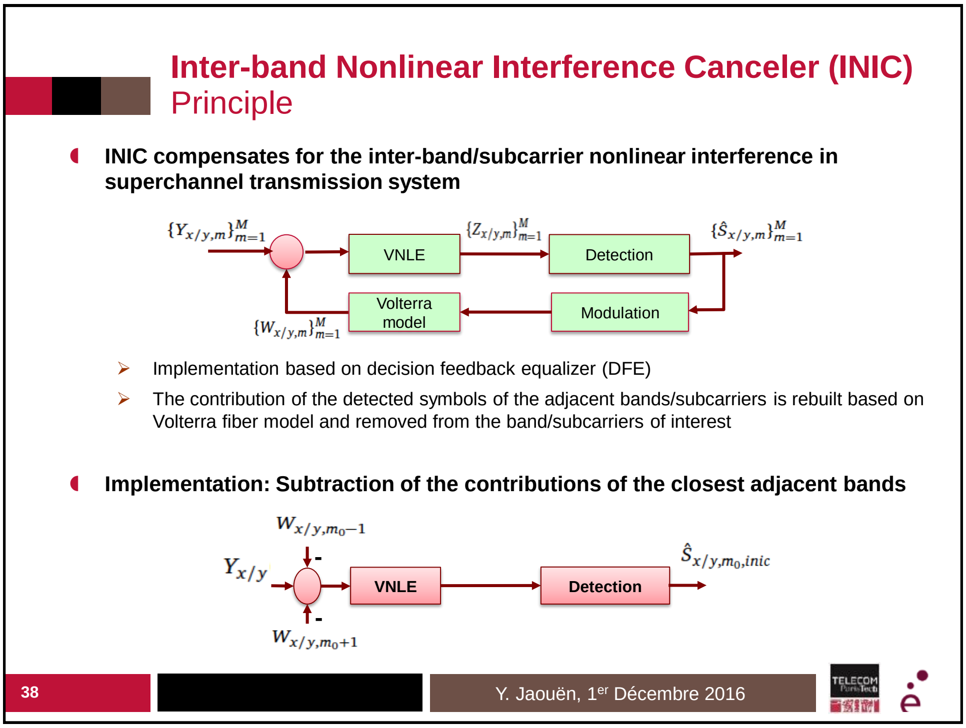
Configuration : 400Gb/s, 4 bands/Channel, 16QAM-OFDM, 10 x 100Km SSMF

- ➢ Single channel case: Inter-band X-talk significantly reduces nonlinear mitigation
- ➢ Maximum Q-factor improvement provided by IVSTF-NLE: 0.6 dB
- ➢ IVSTF demonstrates equal performance with the DBP-SSF_8

V. Vgnopoulou, et al. " Volterra-based Nonlinear Compensation in 400 Gb/s Multiband coherent OFDM Systems" ACP 2014, paper AW1E.4, Nov. 2014

V. Vgnopoulou, et al. " Comparison of multi-channel nonlinear equalization using inverse Volterra series versus digital backpropagation in 400 Gb/s coherent superchannel" ECOC2016, paper Th.2.P2.SC3.31, Sep. 2016

37 Y. Jaouën, 1er Décembre 2016



Inter-band Nonlinear Interference Canceler (INIC)
Principle

- **INIC compensates for the inter-band/subcarrier nonlinear interference in superchannel transmission system**

  - ➢ Implementation based on decision feedback equalizer (DFE)
  - ➢ The contribution of the detected symbols of the adjacent bands/subcarriers is rebuilt based on Volterra fiber model and removed from the band/subcarriers of interest

- **Implementation: Subtraction of the contributions of the closest adjacent bands**

38 Y. Jaouën, 1er Décembre 2016

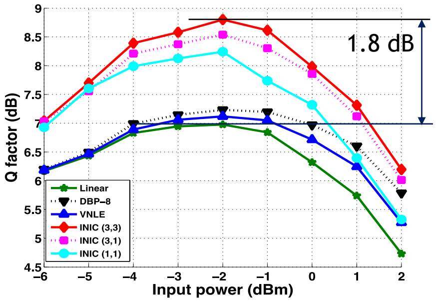
Inter-band Nonlinear Interference Canceler (INIC)

Simulation Results

INIC (3,3) Nonlinear equalization with nonlinear feedback
Equalization based on third-order VNLE
Feedback based on third-order Volterra fiber model

INIC (3,1) Nonlinear equalization with linear feedback
Equalization based on third-order VNLE
Feedback based on linear model

INIC (1,1) Linear equalization with linear feedback



Simulation parameters

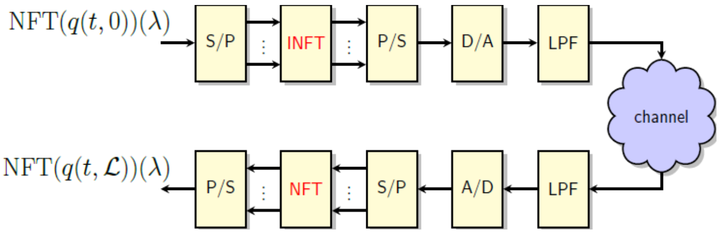
Subcarrier number	4
Bit rate	448 Gbps
Symbol rate	14 GBd
Modulation	**16QAM Nyquist-WDM**
RRC roll off	0.1
Subcarrier spacing	14 GHz ($\Delta f/R_s = 1$)
ADC samples per symbol	2
EDFA noise figure	5.5 dB
Fiber transmission	10 * 100km

The proposed INIC(3,3) and INIC(3,1) strongly outperform VNLE and DBP

K. Amari, et al. " Inter-subcarrier Nonlinear Interference Canceler for Long-Haul Nyquist-WDM Transmission " Photon. Technol. Lett., PTL-28, 2760-2763, December 2016

Nonlinear Fourier Transform Transmission (NFT)

Put information in the **nonlinear spectrum** !



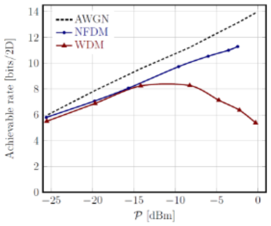
$$NFT(q(t,L))(\lambda) = H(\lambda,L)\ \ x\ \ NFT(q(t,0))(\lambda)$$

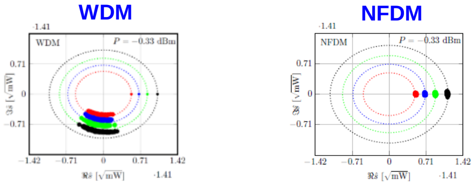
Nonlinear frequency-division multiplexing (NFDM)

M. Yousefi, et al. " Information transmission using the nonlinear Fourier transform, Part I-III" IEEE Trans. Info Theory, vol. 60, no. 7, July 2014

Achievable Data Rates of the WDM and NFDM

B_{opt} = 60 GHz, L = 2000 km
Perfect Raman amplification
(no Loss, Raman ASE)
15 channels, D = -17ps/nm/km
(similar results obtained for D > 0)



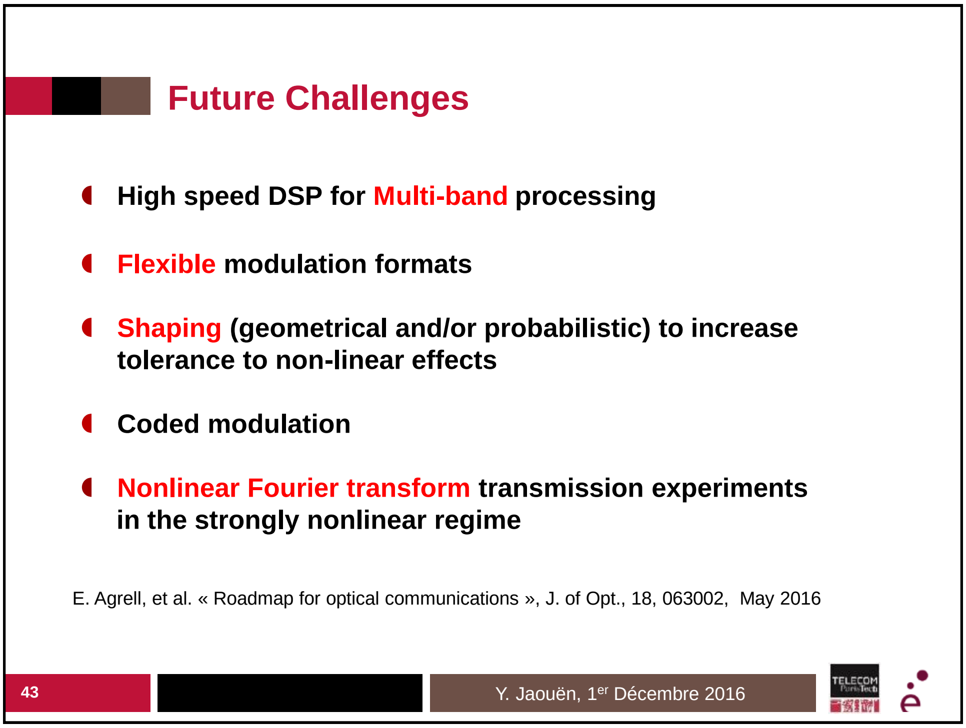


Comparison with the same power and bandwidth

M. Yousefi, et al. " Linear and Nonlinear Frequency-Division Multiplexing" arXiv:1603.04389v2 [cs.IT] 5 May 2016

Conclusions

- **Review of standard coherent DSP**
- **Block-CMA MIMO equalization**
- **Accurate blind frequency offset estimator for QAM**
- **Space-time coding for PDL mitigation**
- **Inter-band nonlinearity canceler**



Future Challenges

- High speed DSP for Multi-band processing
- Flexible modulation formats
- Shaping (geometrical and/or probabilistic) to increase tolerance to non-linear effects
- Coded modulation
- Nonlinear Fourier transform transmission experiments in the strongly nonlinear regime

E. Agrell, et al. « Roadmap for optical communications », J. of Opt., 18, 063002, May 2016